

# 4/MTH-250 Syllabus-2023

2026

( May-June )

**FYUP : 4th Semester Examination**

**MATHEMATICS**

**( Calculus—II )**

( MTH-250 )

*Marks : 75*

*Time : 3 hours*

*The figures in the margin indicate full marks  
for the questions.*

Answer **four** questions, selecting **one** from each Unit.

## UNIT—I

1. (a) If  $f$  is a real-valued function defined on  $[0, 1]$  by  $f(x) = x \forall x \in [0, 1]$ , then prove that  $f$  is Riemann integrable and

$$\int_0^1 f(x) dx = \frac{1}{2}$$

6

( 2 )

- (b) Show that if a real-valued function  $f$  is monotonic in  $[a, b]$ , then it is integrable in  $[a, b]$ .

4

- (c) If  $f$  and  $g$  are two real-valued functions both bounded and integrable in  $[a, b]$  then  $f + g$  is also bounded and integrable in  $[a, b]$  and

$$\int_a^b [f(x) + g(x)] dx = \int_a^b f(x) dx + \int_a^b g(x) dx \quad 3+3=6$$

- (d) Show with the help of an example that the integrability of  $|f|$  may not imply the integrability of the real-valued function  $f$ .

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2. (a) If real-valued function  $f$  is bounded and integrable on  $[a, b]$  and  $m$  and  $M$  be the respective lower and upper bounds of  $f$ , then show that

$$m(b-a) \leq \int_a^b f(x) dx \leq M(b-a) \quad 5$$

- (b) Let  $f$  be a continuous function. Show that the integral function

$$\phi(t) = \int_a^t f(x) dx \text{ is differentiable and } \phi' = f.$$

4

- (c) State and prove Weierstrass' second mean value theorem.

1+5=6

( 3 )

- (d) If a real-valued function  $f$  is bounded and integrable on  $[a, b]$  and  $f(x) \geq 0 \forall x \in [a, b]$ , then show that  $\int_a^b f(x) dx \geq 0$ .

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## UNIT—II

3. (a) Show that if  $f_n(x) = nx e^{-nx^2}$ , the sequence  $\{f_n\}$  is pointwise but not uniformly convergent in  $[0, \infty[$ .

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- (b) Test the convergence of

$$\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}} dx$$

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- (c) Show that  $\int_0^\infty x^{n-1} e^{-x} dx$  is convergent if and only if  $n > 0$ .

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- (d) If  $\{f_n\}$  is a sequence of continuous functions on an interval  $[a, b]$ , and if  $f_n \rightarrow f$  uniformly on  $[a, b]$ , then  $f$  is continuous on  $[a, b]$ . Justify this statement.

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4. (a) Examine the convergence of

$$\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/2}}$$

5

- (b) State and prove Abel's test for uniform convergence of a series of functions.  $1+5=6$
- (c) Prove that  $\sum a_n n^{-x}$ , is uniformly convergent on  $[0, 1]$  if  $\sum a_n$  converges uniformly in  $[0, 1]$ . 4
- (d) Discuss the convergence of  $\int_0^\infty e^{-ax} \cos x dx$ . 4

## UNIT—III

5. (a) Find the value of

$$\int_C (x+y^2)dx + (x^2-y)dy$$

taken in the clockwise sense along the closed curve  $C$  formed by  $y^2 = x$  and  $y = x$  between  $(0, 0)$   $(1, 1)$ . 5

- (b) Express the double integral  $\iint f(x, y) dx dy$  as a repeated integral, where the domain of integration is  $\{(x, y) / x^2 + y^2 \leq a^2\}$ . 4

- (c) Change the order of integration in

$$\int_0^\infty \left\{ \int_x^\infty \frac{e^{-y}}{y} dx \right\} dy$$

and hence find the value. 4

- (d) Evaluate

$$\iint \frac{\sqrt{a^2 b^2 - b^2 x^2 - a^2 y^2}}{\sqrt{a^2 b^2 + b^2 x^2 + a^2 y^2}} dx dy$$

The field of integration being the positive quadrant of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

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6. (a) Find the volume bounded by  $y^2 + z^2 = 4ax$ ,  $y^2 = ax$ ,  $x = 3a$ . 6

- (b) Evaluate

$$\iiint (1-x-y-z)^{l-1} x^{m-1} y^{n-1} z^{p-1} dx dy dz$$

over the interior of the tetrahedron bounded by the planes  $x = 0$ ,  $y = 0$ ,  $z = 0$ ,  $x + y + z = 1$ . 7

- (c) Express the iterated integral

$$\int_0^1 \int_0^{x^2} \int_0^y f(x, y, z) dz dy dx$$

as a triple integral and then rewrite it as an iterated integral in a different order integrating first with respect to  $x$ , then  $z$  and then  $y$ . 6

## UNIT—IV

7. (a) Prove that  $\int \frac{x^2 + y^2}{p} ds$  taken round the

ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  is equal to

$$\frac{1}{4} \pi ab(4 + (a^2 + b^2)(a^{-2} + b^{-2}))$$

where  $p$  is the perpendicular from the centre to the tangent.

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- (b) Evaluate the value of

$$\iint_S p(x^4 + y^4 + z^4) dS$$

where  $S$  is the surface

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

and  $p$  is the perpendicular from the origin to the tangent plane at the point  $(x, y, z)$ .

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- (c) State and prove Stokes' theorem. 1+5=6

8. (a) Verify Green's theorem by evaluating it in two ways the line integral

$$\int_C (x^3 + y^2) dx + (x^2 + y^3) dy$$

taken along the triangle whose vertices are  $(0, 0)$ ,  $(1, 0)$  and  $(2, 1)$ .

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- (b) Apply Stokes' theorem to evaluate

$$\int_C x^2 y^3 dx + dy + z dz$$

where  $C$  is the circle

$$x^2 + y^2 = a^2, z = 0$$

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- (c) Evaluate

$$\iint_S x dy dz + y dz dx + z dx dy$$

taken over the surface of the cube  $[0, a; 0, a; 0, a]$ .

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